FREQUENCY SELECTIVE SURFACES

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Theory and Design

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This book is dedicated to all of my students and colleagues who have taught me so much, and to all my sponsors, who not only taught me but paid me.

This book would not have been possible had it not been for the constant support of my family, and especially that of my wife, Aase.

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FOREWORD I

In the early 1950s the Air Force Avionics Laboratory[1] was involved in efforts to quantify the aircraft characteristic known as the radar cross section (RCS) or radar echoing area which had eluded a solution since WWII. In order to do the calculations and measurements, one had to know how to make an appropriate model, which meant that the features that dominated RCS had to be discovered. In time, a number of these "echo sources" were identified, which subsequently allowed the construction of the needed models and the routine cataloging of RCS of all types of military vehicles.

In the course of these studies, the thought began to grow that it might be possible to reduce the radar size of these prominent echo sources. This started the program on radar signature reduction. The Avionics Lab was the leader in this work of the Department of Defense for many years, but it clearly did not have the manpower to conduct in-house studies and simultaneously look into the treatment of every kind of echo source, which involved long-term development of technology of many types. It was only with the tremendous assistance of a number of organizations that the current state of the art came about.

The first efforts toward echo reduction were devoted to radar absorber materials (RAM) (it seemed reasonable that absorbing incident radar signals from the bad guys was a good thing to do). One particular type of RAM studied was called the circuit analog absorber because it depended on layers of geometric arrays of conductive material which acted like combinations of resistive, capacitive, and inductive circuit components. Chapter 9 of this book treats this topic nicely.

At the same time, one very important echo source was identified that provided an interesting challenge—the large antenna featured so prominently on the nose of most aircraft. The problem was that while we (the good guys) only needed the antenna to

[1]Names of organizations changed frequently over the years, but the most familiar are used herein.

work in a certain small range of frequencies, the bad guys were provided with a big echo source at virtually any frequency they chose to use. Obviously, putting RAM over this could reduce the echo, but we had a feeling that some people on our side might not like the idea too much. It was then (in the late 1950s) that the possibility of using "high Q" (highly conductive) elements, similar to the resistive ones in the circuit analog absorber, was conceived. The idea was to at least severely limit the range of frequencies over which the antenna would be an echo source by substituting a surface that only reflected in a narrow band for the conventional metallic surface that reflected at all frequencies.

At the time the Ohio State University ElectroScience Laboratory was providing major assistance to the Avionics Lab in the RCS activity. The investigation of the "tuned surface" concept was requested because it seemed that a major effort would be required to develop it. At first, progress was slow and not very promising, partly because some of the key players at OSU had doubts about the concept and partly because it was easy to get mired in the complex mathematics (method of moments) chosen as the path to success. After a year or more of frustration, along came Ben Munk, then nearing completion of his doctorate in electromagnetics. To say that Ben was a breath of fresh air would be an understatement.

He immediately took a different tack on the approach, using array theory combined with the detailed method of moments (which became the "Periodic Moment Method" in this book) to handle the problem from a macro rather than a micro view. The results were startling, and the potential of this concept became the source of continued support.

Since that time, Ben has been the worldwide guru of this technology, providing support to applications of all types. His genius lies in handling the extremely complex mathematics while at the same time seeing the practical matters involved in applying the results. As this book clearly shows, Ben is able to relate to novices interested in using frequency selective surfaces and to explain technical details in an understandable way, liberally spiced with his special brand of humor.

Over the years there have been people who were made aware of this work, who proceeded to publish their own ideas on the subject based on "advanced" mathematical analysis. While it is possible to manipulate equations to produce new and interesting theoretical results, most of these publications are of little use. As mentioned earlier, it is in the understanding of the real world design problem that the mathematics produces truly useful results. As this book by Ben Munk clearly demonstrates, he and the team he guided have been able to do just that. His many discussions of "common misconceptions" and little, but very important, factors like "dielectric underwear," serve to demonstrate his mastery of this technology in its most practical form (and his wonderful sense of humor).

Make no mistake about this—Ben Munk has written a book that represents the epitome of practical understanding of frequency selective surfaces. He deserves all honors that might befall him for this achievement. (Continued support for his future work would no doubt be gratefully accepted too.)

William F. Bahret

Mr. W. Bahret was with the United States Air Force but is now retired. From the early 1950s he sponsored numerous projects concerning radar cross section of airborne platforms, in particular antennas and absorbers. Under his leadership grew many of the concepts used extensively today, for example, the metallic radome. In fact he is considered by many to be the father of stealth technology.

B.A.M.

FOREWORD II

As early as 1947 George Sinclair, the founder of the Ohio State University Antenna Laboratory (which later became the ElectroScience Laboratory), realized that antennas placed on aircraft represented significant scatterers. Ed Kennaugh led much of the subsequent research on control of the radar cross section of antennas. This was initiated by a white paper on the "Echoing Area of Antennas" in 1961. One of the concepts introduced for this task was that of frequency selective surfaces, which became the major topic under investigation and which has continued to the present. However, Kennaugh had become deeply involved in transient scattering, complex natural resonances, and the like, and he asked to be replaced as principal investigator on these projects. As a result the laboratory's director asked me to take over this role in 1965. This was a very fortunate event for me in that I became associated with a number of great students. Two of these Ph.D. students, Ben Munk and Randy Ott, focused their research on the analysis of frequency selective surfaces. Both were successful; however, our sponsor (Bill Bahret) commented, "I need two solutions like a turtle needs air brakes!" Randy used more traditional electromagnetic computational moment methods, whereas Ben, from the very beginning of his career, had a very unique viewpoint with respect to this analysis and incorporated the basic physics of interaction between elements into his solution. The four of us (Bill, Ben, Randy, and I) agreed that Ben's approach, which is the same approach he uses in this book, was the most adaptable and thus the most useful. History has demonstrated that the correct choice was made. Since physical concepts are deeply embedded in the analysis of frequency selective surfaces, his approach has subsequently provided useful results for many FSS configurations that weren't even considered in his earlier research. Ben would take over as principal investigator several years after he received the Ph.D. and has remained our FSS guru ever since.

I have a few comments concerning the development of this book. Ben's Danish education has had a strong influence on his writing. His dissertation and subsequent reports were all very well organized. The organization of this book is even better than his earlier works. Also, unfortunately for me, Ben always used a single letter when a double letter occurred in a word, and vice versa. To this day, I sometimes have to go to the dictionary to obtain the correct spelling of certain words. It seems that by the looks of it, Ben has mastered the spelling lesson as the spelling in the present work shows none of this early problem!

This book compiles under one cover most of his research over the past three decades. It is woven with the physical insight that he has gained and further developed as his career has grown. Ben uses mathematics to whatever extent is needed, and only as needed.

This material is written so that it should be useful to engineers with a background in electromagnetics. I strongly recommend this book to any engineer with any interest in phased arrays and/or frequency selective surfaces. The physical insight that may be gained from this book will enhance their ability to treat additional array problems of their own.

LEON PETERS JR.

Leon Peters Jr. was a professor at the Ohio State University but is now retired. From the early 1960s he worked on, among many other things, RCS problems involving antennas and absorbers. In fact he became my supervisor when I joined the group in the mid-1960s.

B.A.M.

PREFACE

The material in this book represents part of the work I performed with my colleagues and students from the mid-1960s to the present time. Much of that work was classified. In fact even my 1968 Ph.D. dissertation became classified. Fortunately, many of these restrictions (but not all) have now been lifted, and I am grateful for this opportunity to publish this book.

Most of the work was sponsored by the U.S. Air Force. However, in the last couple of years funding has primarily come from the U.S. Navy, which has actually made this book possible.

BEN MUNK

November 1999

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Many individuals have supported and inspired me over the years. To list all of them would be impossible, but certain people stand out. Foremost are Mr. William F. Bahret, Dr. Leon Peters, Jr., and Dr. Robert Kouyoumjian. Without a doubt, they are the three individuals who had the greatest influence on my work in stealth technology. Later, they were joined or followed by Dr. Carl Mentzer, Dr. Brian Kent, and Mr. Ed Utt.

While the U.S. Air Force primarily was interested in band pass filters, the U.S. Navy typically was more interested in band stop designs. Immense, valuable guidance and support was given to me by Mr. Jim Logan, Dr. John Meloling, and Dr. John Rockway.

My gratitude is also extended to my good friend Mr. Clyde L. Hoots from the Marion Composite (formerly Brunswick) Corporation. He taught me many practical aspects of radome design.

I am also indebted to my friend and colleague Prof. John Kraus for the constant encouragement and advice he has given me over the years.

Numerous students have assisted me over the years and many of them are referenced in the text. A special thank you goes to Mr. Ethan Saladin, Mr. Jonothan Pryor, and Mr. Dan Janning for running many of the computed cases. I am further indebted to the students in my last EE910 class. They corrected numerous errors and made many good suggestions.

Finally, my deep felt thanks goes to Miss Ann Kelly who typed the entire manuscript and to Mr. Jim Gibson who drafted all the manually-generated drawings. They both did a superb job.

B.A.M.

SYMBOLS AND DEFINITIONS

a	Horizontal distance between column q and point of observation $\bar{R}$
a, a_1	Wire radius of elements
a	Side length of square elements
$d\bar{A}$	Vector potential for double infinite array of Hertzian elements
$d\bar{A}_q$	Vector potential of Hertzian elements located in column q
$d\bar{A}_{qm}$	Vector potential of a single Hertzian element located in column q and row m
b_{m-1}	Location of the front face of dielectric slab m in dipole case
b_m	Location of the back face of dielectric slab m in dipole case
C_p	Equivalent shunt capacitance from the orthogonal elements in a circuit analog absorber
d	Diameter of circular plate element
d_m	Thickness of dielectric slab in dipole case
D_N	Determinant of admittance matrix for N slot arrays

D_x	Inter-element spacings in the x-direction
D_z	Inter-element spacings in the z-direction
$\bar{e} = \left[\hat{p} \times \hat{r}\right] \times \hat{r} = {}_{\perp}\hat{n}_{\perp}e + {}_{\parallel}\hat{n}_{\parallel}e$	Field vector for infinite array of Hertzian elements
$\bar{E}_m(\bar{R})$	Electric field at $\bar{R}$ in medium m
$\bar{E}^i_m(\bar{R})$	Incident electric field at $\bar{R}$ in medium m
$\bar{E}^r_m(\bar{R})$	Reflected electric field at $\bar{R}$ in medium m
f	Frequency
f_g	Onset frequency of grating lobe
$F(w)$	Fourier transform of $f(t)$, not necessarily a function of time
$\bar{H}_m(\bar{R})$	Magnetic field of $\bar{R}$ in medium m
$\bar{H}^i_m(\bar{R})$	Incident magnetic field at $\bar{R}$ in medium m
$\bar{H}^r_m(\bar{R})$	Reflected magnetic field at $\bar{R}$ in medium m
$H_n^{(2)}(x)$	Hankel's function of the second kind, order n and argument x
$I_{qm}(l)$	Current along element in column q and row m
k, n	Indexes for the spectrum of plane, inhomogeneous waves from an infinite array
l	Distance from a reference point to an arbitrary point on the element
$2l_1$	Total element length
dl	Infinitesimal element length
Δl	Element length of Hertzian dipoles
$\bar{m}_{\pm} = \bar{E} \times \hat{n}_{D\pm}$	Magnetic current density
$\bar{M}_{\pm}$	Total magnetic current in slots
$\hat{n}_D$	Unit vector orthogonal to dielectric interface pointing into the dielectric medium in question
${}_{\perp}\hat{n}_m = \frac{\hat{n}_D \times \hat{r}}{\lvert n_D \times \hat{r} \rvert}$	Unit vector(s) orthogonal to the planes of incidence or reradiation in medium m
${}_{\parallel}\hat{n}_m = {}_{\perp}\hat{n}_m \times \bar{r}$	Unit vector(s) parallel to the planes of incidence or reradiation in medium m.
$n, n_0, n_1, n_2, \ldots$	Integers

$\hat{p}$	Orientation vector for elements
$\hat{p}^{(p)}$	Orientation vector for element section p
$\hat{p}^{p,n}$	Orientation vector for element section p in array n
$P^{(p)}$	Scattering pattern function associated with element section p
$P^{(p)t}$	Transmitting pattern function associated with element section p
$P_m^{(p)}$	Scattering pattern function associated with element section p in medium m
$\underset{\parallel}{\perp} P_{m\pm}^{(p)} = \hat{p}^{(p)} \cdot \underset{\parallel}{\perp} \hat{n}_{m\pm} P_{m\pm}^{(p)}$	Orthogonal and parallel pattern components of scattering pattern in medium m
$P_m^{(p)t}$	Transmitting pattern function associated with element section p in medium m
$\underset{\parallel}{\perp} P_m^{(p)t} = \hat{p}^{(p)} \cdot \underset{\parallel}{\perp} \hat{n}_{m\pm} P_{m\pm}^{(p)t}$	Orthogonal and parallel pattern components of transmitting pattern in medium m
$\mathcal{P}_n$	Polynomial for a band-pass filter comprised of n slot arrays
q, m	Position of a single element in column q and row m
$\hat{r}_\pm = \hat{x} r_x \pm \hat{y} r_y + \hat{z} r_z$	Direction vectors of the plane wave spectrum from an infinite array
$\hat{r}_{m\pm} = \hat{x} r_{mx} \pm \hat{y} r_{my} + \hat{z} r_{mz}$	Direction vectors in medium m of the plane wave spectrum from an infinite array
$r_\rho = \sqrt{1 - \left(s_z + n\frac{\lambda}{D_z}\right)^2}$	ρ-Component of $\hat{r}_\pm$
$\hat{s} = \hat{x} s_x + \hat{y} s_y + \hat{z} s_z$	Direction of incident field
$\hat{s}_m = \hat{x} s_{mx} + \hat{y} s_{my} + \hat{z} s_{mz}$	Direction of incident field in medium m
t	Variable used in Poisson's sum formula
$\underset{\parallel}{\perp} T_m$	Orthogonal and parallel transformation functions for single dielectric slab of thickness d_m
$\underset{\parallel}{\overset{E}{\perp}} T_m$	Orthogonal and parallel transformation function for the E-field in a single dielectric slab of thickness d_m
$\underset{\parallel}{\overset{H}{\perp}} T_m$	Orthogonal and parallel transformation function for the H-field in a single dielectric slab of thickness d_m

${}_{\parallel}^{\perp} T_{m-m'}$	Orthogonal and parallel generalized transformation function when going from one dielectric slab of thickness d_m to another of thickness $d_{m'}$, both of which are located in a general stratified medium
$T.C._{\pm 1}$	Transmission coefficient at the roots $Y_{1\pm}$, etc.
$V^{1',1}$	Induced voltage in an external element with reference point $\bar{R}^{(1')}$ caused by all the currents from an array with reference element at $\bar{R}^{(1)}$
$V_{Di\pm}^{(1')}$	Induced voltage in an external element with reference point $\bar{R}^{(1')}$ caused by a direct wave only from the entire array
$V_{D\pm}^{(1')}$	Induced voltage in an external element with reference point $\bar{R}^{(1')}$ caused by double bounced modes ending in the $\pm$ direction
$V_{S\pm}^{(1')}$	Induced voltage in an external element with reference point $\bar{R}^{(1')}$ caused by a single bounded mode ending up in the $\pm$ direction
w	Dipole or slot width
${}_{\parallel}^{\perp} W_m$	Orthogonal and parallel components for the Wronskian for a single dielectric slab of thickness d_m
${}_{\parallel}^{\perp} W_m^e$	Orthogonal and parallel components for the effective Wronskian for a single dielectric slab of thickness d_m and located in a general stratified medium
Y	Intrinsic admittance
y	Variable defined by (6.29)
$y_{1\pm}, y_{2\pm}, \ldots$	Roots of polynomial for band-pass filter
Y_A	Scan admittance as seen at the terminals of an element in the array
Y_L	Load admittance at the terminals of the elements
$Y_0 = \frac{1}{Z_0}$	Intrinsic admittance of free space
$Y_m = 1/Z_m$	Intrinsic admittance of medium m
$Y^{1,2}$	Array mutual admittance between array 1 and 2
Z	Intrinsic impedance

$Z = \frac{a+bz}{c+dz}$	Dependent variable as a function of the independent variable z in a bilinear transformation
$Z_0 = 1/Y_0$	Intrinsic impedance of free space
$Z_A = R_A + jX_A$	Scan impedance as seen at the terminals of an element in the array
Z_L	Load impedance at the terminals of the elements
$Z_m = 1/Y_m$	Intrinsic impedance of medium m
$Z^{n,n'}$	Array mutual impedance between a reference element in array n and double infinite array n'
$Z^{q,q'}$	Column mutual impedance between a reference element in column q and an infinite line array at q'
$Z_{q,q'm}$	Mutual impedance between reference element in column q and element m in column q'
α	Angle between plane of incidence and the xy-plane
$\beta_m = \frac{2\pi}{\lambda_m}$	Propagation constant in medium m.
Δl	Total element length of Hertzian dipole
ϵ	Dielectric constant
ϵ_{eff}	Effective dielectric constant of a thin dielectric slab as it affects the resonant frequency
ϵ_m	Dielectric constant in medium m
ϵ_{rm}	Relative dielectric constant in medium m
η	Angle of incidence from broadside
η_g	Angle of grating lobe direction from broadside
θ_m	Angle of incidence from broadside in medium m
${}^{E}_{\perp\atop\parallel}\Gamma_{m+} = {}^{E}_{\perp\atop\parallel}\Gamma_{m,m+1}$	Orthogonal and parallel Fresnel reflection coefficient for the E-field when incidence is from media m to $m+1$
${}^{H}_{\perp\atop\parallel}\Gamma_{m+} = {}^{H}_{\perp\atop\parallel}\Gamma_{m,m+1}$	Orthogonal and parallel Fresnel reflection coefficient for the H-field when incidence is from media m to $m+1$
${}^{E}_{\perp\atop\parallel}\Gamma^{e}_{m+} = {}^{E}_{\perp\atop\parallel}\Gamma^{e}_{m+1}$	Orthogonal and parallel effective reflection coefficient for the E-field when incidence is form media m to $m+1$

${}^{H}_{\perp \atop \parallel}\Gamma^{e}_{m+} = {}^{H}_{\perp \atop \parallel}\Gamma^{e}_{m,m+1}$	Orthogonal and parallel effective reflection coefficient for the H-field when incidence is from media m to $m + 1$
λ_m	Wavelength in medium m
μ_m	Permeability in medium m
μ_{rm}	Relative permeability in medium m
${}^{E}_{\perp \atop \parallel}\tau_{m+} = {}^{E}_{\perp \atop \parallel}\tau_{m,m+1}$	Orthogonal and parallel Fresnel transmission coefficient for the E-field when incidence is from media m to $m + 1$
${}^{H}_{\perp \atop \parallel}\tau_{m+} = {}^{H}_{\perp \atop \parallel}\tau_{m,m+1}$	Orthogonal and parallel Fresnel transmission coefficient for the H-field when incidence is from media m to $m + 1$
${}^{E}_{\perp \atop \parallel}\tau^{e}_{m+} = {}^{E}_{\perp \atop \parallel}\tau^{e}_{m,m+1}$	Orthogonal and parallel effective transmission coefficient for the E-field when incidence is from media m to $m + 1$
${}^{H}_{\perp \atop \parallel}\tau^{e}_{m+} = {}^{H}_{\perp \atop \parallel}\tau^{e}_{m,m+1}$	Orthogonal and parallel effective transmission coefficient for the H-field when incidence is from media m to $m + 1$
$\omega = 2\pi f$	Angular frequency
$\omega_1\omega_0$ and ω_1	Variables used in Poisson's sum formula (not angular frequencies)